

# Pion form factor and QCD sum rules: case of axial current.

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## Abstract

We present an analysis of QCD sum rules for pion form factor in next-to-leading order of perturbation theory for the case of axial-vector pion currents. The theoretical predictions for  $Q^2$ -dependence of pion form factor are in good agreement with available experimental data. It is shown, that NLO corrections are large in this case and should be taken into account in rigorous theoretical analysis.

## 1 Introduction

The study of electromagnetic form factors of hadrons already has a long history. The first applications appeared right after the realization, that perturbative QCD may be also applied in studies of exclusive processes with high momentum transfer [1, 2, 3]. However, later comparison of pQCD predictions with data lead to conclusion, that at moderate momentum transfers (typically of the order of few GeV) the power corrections, otherwise known as soft or end-point contributions should come into play<sup>1</sup>.

In this paper we are using the framework of QCD sum rules [4] to consistently account for both hard scattering and soft wave function overlap contributions to pion electromagnetic form factor. Within this approach the soft contribution is dual to the lowest-order triangle diagram, while the hard contribution is given by diagrams having higher order in coupling constant  $\alpha_s$  and as a consequence suppressed relative to soft contribution with additional factor  $\alpha_s/\pi \sim 0.1$ . This extra suppression is in a complete agreement with asymptotic behavior of the pion electromagnetic form factor, calculated in pQCD [1, 2, 3]:

$$F_{\pi}^{\text{hard}}(Q^2) = \frac{8\pi\alpha_s(Q^2)}{9} \int_0^1 dx \int_0^1 dy \frac{\phi_{\pi}(x)\phi_{\pi}(y)}{xyQ^2} = \frac{8\pi\alpha_s f_{\pi}^2}{Q^2}, \quad (1)$$

where the last equality holds for the asymptotic pion distribution amplitude. At asymptotically high  $Q^2$  the  $\mathcal{O}(\alpha_s/\pi)$  suppression of hard contribution is more than compensated by its slower decrease with  $Q^2$ . However, such compensation does not occur in the region of moderate momentum transfer, where the soft contribution, scaling as  $1/Q^4$  becomes large and can compete in strength with hard contribution.

The pion electromagnetic form factor was studied using many different frameworks, like QCD sum rules [6, 7, 8, 9], light-cone sum rules [10, 11, 12], sum rules with nonlocal condensates [13] and NLO pQCD approach [14, 15, 16, 17, 18], improved by inclusion of finite size corrections

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<sup>1</sup>See, for example, discussion in [5].

to pion distribution function [19, 20, 21]. There are also estimates of pion electromagnetic form factor based on use of pseudoscalar pion interpolating currents [22, 23, 24].

In what follows we will consider NLO three-point QCD sum rules for pion form factor, where axial currents are used as pion interpolating currents. The main result of this paper is the explicit analytical expression for QCD radiative corrections to double spectral density, entering QCD sum rule predictions for pion electromagnetic form factor. It should be noted, that up to the moment there are only radiative corrections computed for reduced spectral density [25, 26].

The obtained results for pion electromagnetic form factor calculation are in good agreement with existing experimental data. Here, we would like to stress that an inclusion of radiative corrections is very important, as only in such a way we can simultaneously account for both hard and soft contributions. Moreover, these corrections are large numerically and thus should be accounted for in theoretical predictions.

The paper is organized as follows. In section 2 we describe our framework and give explicit expressions for next-to-leading order corrections to double spectral density. Section 3 contains our numerical analysis. Finally, in section 4 we draw our conclusions.

## 2 Derivation of QCD sum rules

To determine pion electromagnetic form factor we will use the method of three-point QCD sum rules. Here, to describe charged pion state we choose an action of axial interpolating current on a vacuum state. The vacuum to pion transition matrix element of axial current is defined by

$$\langle 0 | \bar{u} \gamma_5 \gamma_\mu d | \pi^-(p) \rangle = i f_\pi p_\mu, \quad (2)$$

where  $f_\pi = 131$  MeV. Next, the pion electromagnetic form factor, we are planning to determine, is given by hadronic matrix element of electromagnetic current:

$$\langle \pi(p') | j_\mu^{\text{el}} | \pi(p) \rangle = F_\pi(Q^2)(p_\mu + p'_\mu), \quad (3)$$

where  $j_\mu^{\text{el}} = e_u \bar{u} \gamma_\mu u + e_d \bar{d} \gamma_\mu d$ , the momenta of initial and final state pions were denoted by  $p$ ,  $p'$  and  $Q^2 = -q^2$  ( $q = p - p'$ ) is square of momentum transfer.

Within the framework of QCD sum rules an expression for pion electromagnetic form factor follows from an analysis of corresponding three-point correlation function:

$$\Pi_{\mu\alpha\beta}(p, p', q) = i^2 \int dx dy e^{i(p' \cdot x - p \cdot y)} \langle 0 | T \{ \bar{u}(x) \gamma_5 \gamma_\alpha d(x), j_\mu^{\text{el}}(0), (\bar{u}(y) \gamma_5 \gamma_\beta d(y))^+ | 0 \rangle \quad (4)$$

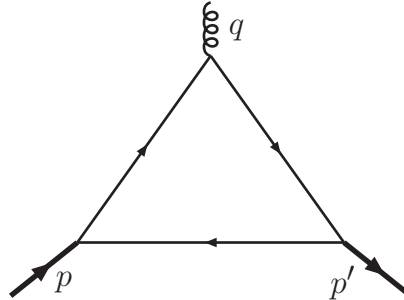


Figure 1: LO diagram

This correlation function contains a lot of different tensor structures. The scalar amplitudes  $\Pi_i$  in front of different Lorentz structures are the functions of kinematical invariants, i.e.  $\Pi_i =$

$\Pi_i(p^2, p'^2, q^2)$ . The calculation of QCD expression for three-point correlator is done through the use of operator product expansion (OPE) for the T-ordered product of currents. As a result of OPE one obtains besides leading perturbative contribution also power corrections, given by vacuum QCD condensates. We will return to the discussion of QCD expression for three-point correlation function in a moment. Now let us discuss the physical part of our sum rules. The connection to hadrons in the framework of QCD sum rules is obtained by matching the resulting QCD expressions for current correlators with spectral representation, following the structure of double dispersion relation at  $q^2 \leq 0$ :

$$\Pi_{\mu\alpha\beta}(p_1^2, p_2^2, q^2) = \frac{1}{(2\pi)^2} \int \frac{\rho_{\mu\alpha\beta}^{\text{phys}}(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} ds_1 ds_2 + \text{subtractions.} \quad (5)$$

Assuming that the dispersion relation (5) is well convergent, the physical spectral functions are generally saturated by the lowest lying hadronic states plus a continuum starting at some thresholds  $s_1^{\text{th}}$  and  $s_2^{\text{th}}$ :

$$\begin{aligned} \rho_{\mu\alpha\beta}^{\text{phys}}(s_1, s_2, Q^2) &= \rho_{\mu\alpha\beta}^{\text{res}}(s_1, s_2, Q^2) + \\ &\theta(s_1 - s_1^{\text{th}}) \cdot \theta(s_2 - s_2^{\text{th}}) \cdot \rho_{\mu\alpha\beta}^{\text{cont}}(s_1, s_2, Q^2), \end{aligned} \quad (6)$$

where

$$\begin{aligned} \rho_{\mu\alpha\beta}^{\text{res}}(s_1, s_2, Q^2) &= \langle 0 | \bar{u} \gamma_5 \gamma_\alpha d | \pi^-(p') \rangle \langle \pi^-(p') | j_\mu^{\text{el}} | \pi^-(p) \rangle \langle \pi^-(p) | (\bar{u} \gamma_5 \gamma_\beta d)^+ | 0 \rangle \cdot \\ &(2\pi)^2 \delta(s_1) \delta(s_2) + \text{higher state contributions} \end{aligned} \quad (7)$$

In our approximation of massless quarks we put  $m_\pi^2 = 0$ . So we see, that pion contribution into spectral density is given by  $\rho_{\mu\alpha\beta}^{\text{pion}} \sim f_\pi^2 F_\pi(Q^2) p^\alpha p'^\beta (p^\mu + p'^\mu)$  and as was already noted in [6, 7, 8, 9] the most convenient way to extract pion form factor from QCD sum rules is to consider scalar amplitude in front of most symmetric Lorentz structure  $P_\mu P_\alpha P_\beta$  ( $P = p + p'$ ).

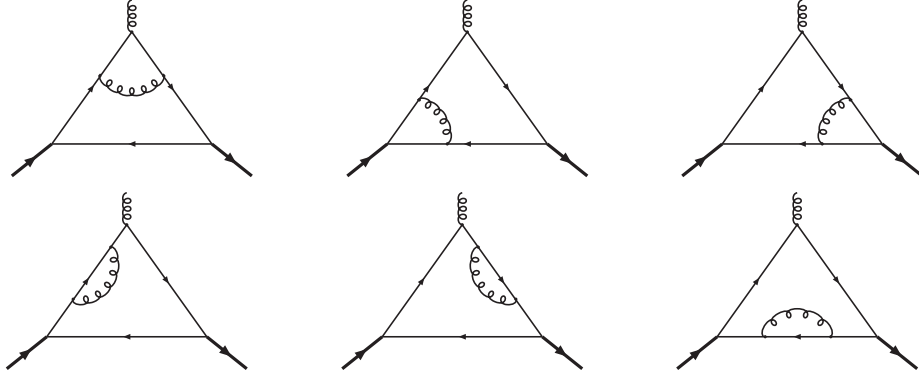


Figure 2: NLO diagrams

Now it is time to return to the problem of evaluation of QCD expression for three point correlation function, we are interested in. The condensate contribution is known already for a long time [6, 7, 8, 9] and its analytical expression could be found in the section with numerical results. The calculation of perturbative contribution could be conveniently performed with the use of double dispersion representation in variables  $s_1 = p^2$  and  $s_2 = p'^2$  at  $q^2 < 0$ :

$$\Pi_{\mu\alpha\beta}^{\text{pert}}(p^2, p'^2, q^2) = \frac{1}{(2\pi)^2} \int \frac{\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2)}{(s_1 - p^2)(s_2 - p'^2)} ds_1 ds_2 + \text{subtractions} \quad (8)$$

The integration region in (8) is determined by condition <sup>2</sup>

$$-1 \leq \frac{s_2 - s_1 - q^2}{\lambda^{1/2}(s_1, s_2, q^2)} \leq 1 \quad (9)$$

and

$$\lambda(x_1, x_2, x_3) = (x_1 + x_2 - x_3)^2 - 4x_1x_2. \quad (10)$$

The double spectral density  $\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2)$  is searched in the form of expansion in strong coupling constant:

$$\rho_{\mu\alpha\beta}^{\text{pert}}(s_1, s_2, Q^2) = \rho_{\mu\alpha\beta}^{(0)}(s_1, s_2, Q^2) + \left(\frac{\alpha_s}{4\pi}\right) \rho_{\mu\alpha\beta}^{(1)}(s_1, s_2, Q^2) + \dots \quad (11)$$

At leading order in coupling constant we have only one diagram depicted in Fig. 1, contributing to three-point correlation function. At next to leading order we have 6 diagrams shown in Fig. 2. The calculation of corresponding double spectral density was performed with the standard use of Cutkosky rules. In the kinematic region  $q^2 < 0$ , we are interested in, there is no problem in applying Cutkosky rules for determination of  $\rho_{\mu\alpha\beta}(s_1, s_2, Q^2)$  and integration limits in  $s_1$  and  $s_2$ . The non-Landau type singularities, not accounted for by Cutkosky prescription, do not show up here.

It is easy to find, that at Born level the scalar spectral density in front of most symmetric Lorentz structure  $P_\mu P_\alpha P_\beta$  is given by:

$$\begin{aligned} \rho_{\mu\alpha\beta}^{(0)}(s_1, s_2, Q^2) = & \frac{3Q^4}{4} \frac{1}{k^{7/2}} \left( 3k(s_1 + s_2 + Q^2)(s_1 + s_2 + 2Q^2) - k^2 \right. \\ & \left. - 5Q^2(s_1 + s_2 + Q^2)^3 \right) P_\mu P_\alpha P_\beta + \dots, \quad (12) \end{aligned}$$

where  $k = \lambda(s_1, s_2, -Q^2)$ . The full analytical expression for  $\rho_{\mu\alpha\beta}^{(0)}$  could be found in [7]. The calculation of NLO radiative corrections to double spectral density is in principle straightforward. One just needs to consider all possible double cuts of diagrams, shown in Fig. 2. However, the presence of collinear and soft infrared divergences calls for appropriate regularization of arising divergences at intermediate steps of calculation and makes the whole analytical calculation quite involved. We will present the details of NLO calculation in one of our future publications. Here we give only final results. The calculation could be considerably simplified with the help of Lorentz decomposition of double spectral density based on a fact, that our spectral density is subject to three transversality conditions:  $\rho_{\mu\alpha\beta} q_\mu = \rho_{\mu\alpha\beta} p_\alpha = \rho_{\mu\alpha\beta} p'_\beta = 0$ :

$$\begin{aligned} \rho^{\mu\alpha\beta} = & A_1[(Q^2 + x)p_1^\alpha - (x + y)p_2^\alpha][(y - x)p_1^\beta + (Q^2 + x)p_2^\beta][(Q^2 + y)p_1^\mu + (Q^2 - y)p_2^\mu] \\ & - \frac{1}{2}A_2[(Q^2 + y)p_1^\mu + (Q^2 - y)p_2^\mu][(Q^2 + x)g^{\alpha\beta} - 2p_1^\beta p_2^\alpha] \\ & - \frac{1}{2}A_3[(Q^2 + x)p_1^\alpha - (x + y)p_2^\alpha][2(p_2^\beta - p_1^\beta)p_2^\mu + (Q^2 + y)g^{\mu\beta}] \\ & - \frac{1}{2}A_4[(x - y)p_1^\beta - (Q^2 + x)p_2^\beta][2(p_2^\alpha - p_1^\alpha)p_1^\mu + (y - Q^2)g^{\mu\alpha}], \quad (13) \end{aligned}$$

where  $x = s_1 + s_2$  and  $y = s_1 - s_2$ . The four independent structures  $A_i$  (we suppressed the dependence on kinematical invariants) are given by a solution of system of linear equations:

$$I_1 = \rho_{\mu\alpha\beta} p_1^\mu p_2^\alpha p_1^\beta = \frac{k^2}{8} (kA_1 - A_2 - A_3 - A_4) \quad (14)$$

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<sup>2</sup>In our case this inequality is satisfied identically.

$$I_2 = \rho_{\mu\alpha\beta} p_1^\mu g^{\alpha\beta} = \frac{k}{4}(x + Q^2) \left( kA_1 - 3A_2 - A_3 - A_4 \right) \quad (15)$$

$$I_3 = \rho_{\mu\alpha\beta} p_2^\alpha g^{\mu\beta} = \frac{k}{4}(y + Q^2) \left( kA_1 - A_2 - 3A_3 - A_4 \right) \quad (16)$$

$$I_4 = \rho_{\mu\alpha\beta} p_1^\beta g^{\mu\alpha} = -\frac{k}{4}(y - Q^2) \left( kA_1 - A_2 - A_3 - 3A_4 \right), \quad (17)$$

The analytical expressions for  $I_i$  (functional dependence on kinematical invariants is assumed) were found to be ( $s_3 = Q^2$ ):

$$\begin{aligned} k^{1/2} I_1 = & -s_1^3 + s_2 s_1^2 + s_2^2 s_1 - s_2^3 + (s_1 + s_2) s_3^2 - s_1^2 s_3 - s_2^2 s_3 + s_1 s_2 s_3 \left[ \right. \\ & -16 \log^2(v_1) - 16 \log(v_3) \log(v_1) - 16 \log(v_4) \log(v_1) \\ & + 2 \log^2(v_1) - 4 \log^2(v_3) - 4 \log^2(v_4) - 2 \log(v_2) - 2 \log(v_3) - 8 \log(v_3) \log(v_4) \\ & \left. - 8 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 8 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 8 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 8 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 8 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (18) \end{aligned}$$

$$\begin{aligned} k^{1/2} I_2 = & -2s_1^2 - 2s_2^2 + 2s_3^2 - 8s_1 s_2 + s_1 s_2 \left[ \right. \\ & -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\ & -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\ & \left. - 16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (19) \end{aligned}$$

$$\begin{aligned} k^{1/2} I_3 = & -2s_1^2 + 2s_2^2 + 2s_3^2 - 8s_2 s_3 + s_2 s_3 \left[ \right. \\ & -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\ & -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\ & \left. - 16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (20) \end{aligned}$$

$$\begin{aligned} k^{1/2} I_4 = & 2s_1^2 - 2s_2^2 + 2s_3^2 - 8s_1 s_2 + s_1 s_3 \left[ \right. \\ & -32 \log^2(v_1) - 32 \log(v_3) \log(v_1) - 32 \log(v_4) \log(v_1) + 4 \log(v_1) \\ & -8 \log^2(v_3) - 8 \log^2(v_4) - 4 \log(v_2) - 4 \log(v_3) - 16 \log(v_3) \log(v_4) \\ & \left. - 16 \text{Li}_2\left(\frac{x_2}{x_1}\right) - 16 \text{Li}_2\left(\frac{y_1}{y_2}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_1}\right) - 16 \text{Li}_2\left(\frac{z_1}{s_2}\right) + 16 \text{Li}_2\left(\frac{z_1}{z_2}\right) \right], \quad (21) \end{aligned}$$

where the following notation was introduced:

$$x_1 = \frac{1}{2}(s_1 - s_2 - Q^2) - \frac{1}{2}\sqrt{k}, \quad (22)$$

$$x_2 = \frac{1}{2}(s_1 - s_2 - Q^2) + \frac{1}{2}\sqrt{k}, \quad (23)$$

$$y_1 = \frac{1}{2}(s_1 + Q^2 - s_2) - \frac{1}{2}\sqrt{k}, \quad (24)$$

$$y_2 = \frac{1}{2}(s_1 + Q^2 - s_2) + \frac{1}{2}\sqrt{k}, \quad (25)$$

$$z_1 = \frac{1}{2}(s_1 + s_2 + Q^2) - \frac{1}{2}\sqrt{k}, \quad (26)$$

$$z_2 = \frac{1}{2}(s_1 + s_2 + Q^2) + \frac{1}{2}\sqrt{k}, \quad (27)$$

$$v_1 = \frac{1}{2s_1}(s_1 - s_2 - Q^2) + \frac{1}{2s_1}\sqrt{k}, \quad (28)$$

$$v_2 = \frac{1}{2s_2}(s_1 - s_2 + Q^2) + \frac{1}{2s_2}\sqrt{k}, \quad (29)$$

$$v_3 = \frac{1}{2s_1}(s_1 + s_2 + Q^2) + \frac{1}{2s_1}\sqrt{k}, \quad (30)$$

$$v_4 = \frac{s_1}{Q^2}, \quad (31)$$

$$v_5 = \frac{s_2}{Q^2}, \quad (32)$$

$$v_6 = 1 - \frac{z_1}{z_2}. \quad (33)$$

We checked, that all infrared and ultraviolet divergences cancel as should be for axial interpolating currents. Finally, NLO scalar spectral density in front of most symmetric Lorentz structure  $P_\mu P_\alpha P_\beta$  is

$$\begin{aligned} \rho_{\mu\alpha\beta}^{(1)} = & \frac{Q^2}{k^3} \left\{ \frac{1}{2}(x_1 + x_2)kI_3 + (k - 5(x_1 + x_2)(y_1 + y_2))I_1 \right. \\ & \left. + \frac{1}{2}(y_1 + y_2)kI_4 + \frac{1}{2} \frac{k}{z_1 + z_2} ((x_1 + x_2)(y_1 + y_2) - k)I_2 \right\} P_\mu P_\alpha P_\beta + \dots \end{aligned} \quad (34)$$

In the limit  $Q^2 \rightarrow \infty$  our NLO double spectral density takes the following form:

$$\rho_{\mu\alpha\beta}^{(1)} = \left\{ \frac{2}{Q^2} - 10 \frac{s_1 + s_2}{Q^4} - 2 \frac{s_1 + s_2}{Q^4} \log \left( \frac{s_1 s_2}{Q^4} \right) \right\} P_\mu P_\alpha P_\beta + \dots \quad (35)$$

Here we would like to make several comments. First, we see that double logarithms are absent in our answer. Typically we would expect them to be nonzero as diagrams in Fig.2 contain Sudakov vertex - corrections to  $q$ -vertex. And indeed our result for gluon correction to electromagnetic vertex (accompanied by the appropriate  $1/2$  self-energy insertions to ensure ultraviolet-finite result) contains double logs and agrees with large  $Q^2$  limit of [25, 26]. However, our results for corrections to  $p_1$  and  $p_2$  vertexes also contain double logs (an analogue of double logs arising in pQCD description of pion electromagnetic form factor [19, 20]). In the sum of all diagrams double logs cancel. Second, using large  $Q^2$  expression for our spectral density and continuum subtraction thresholds for  $s_1$  and  $s_2$  equal to  $4\pi^2 f_\pi^2$  it is easy to see that leading  $Q^2 \rightarrow \infty$  QCD sum rule expression for pion electromagnetic form factor is given by leading pQCD prediction (1) with asymptotic pion distribution function ( $\phi_\pi^{as}(x) = 6f_\pi x(1-x)$ ). Third, we checked numerically, that a  $Q^2 \rightarrow 0$  limit of our spectral density do satisfy Ward identity constraints<sup>3</sup> [27]. Now let us proceed with numerical analysis.

### 3 Numerical analysis

In numerical analysis we used Borel scheme of QCD sum rules. That is, to get rid of unknown subtraction terms in (8) we perform Borel transformation procedure in two variables  $s_1$  and  $s_2$ . The Borel transform of three-point function  $\Pi_i(s_1, s_2, q^2)$  is defined as

$$\begin{aligned} \Phi(M_1^2, M_2^2, q^2) \equiv & \hat{B}_{12} \Pi_i(s_1, s_2, q^2) = \\ & \lim_{n, m \rightarrow \infty} \left\{ \frac{s_2^{n+1}}{n!} \left( -\frac{d}{ds_2} \right)^n \frac{s_1^{m+1}}{m!} \left( -\frac{d}{ds_1} \right) \Big|_{s_1=mM_1^2, s_2=nM_2^2} \right\} \Pi(s_1, s_2, q^2) \end{aligned} \quad (36)$$

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<sup>3</sup>We are grateful to A.P.Bakulev for explaining to us the details of both this limit and Ward identity constraints for double spectral density

Then Borel transformation (36) of (8) and (5) gives

$$\Phi^{(\text{pert}|\text{phys})}(M_1^2, M_2^2, q^2) = \frac{1}{(2\pi)^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \exp \left[ -\frac{s_1}{M_1^2} - \frac{s_2}{M_2^2} \right] \rho^{(\text{pert}|\text{phys})}(s_1, s_2, q^2), \quad (37)$$

where  $\rho^{(\text{pert}|\text{phys})}(s_1, s_2, q^2)$  stands for the expression of scalar spectral density in front of  $P_\mu P_\alpha P_\beta$  Lorentz structure. In what follows we put  $M_1^2 = M_2^2 = M^2$ . If  $M^2$  is chosen to be of order 1  $\text{GeV}^2$ , then the right hand side of (37) in the case of physical spectral density will be dominated by the lowest hadronic state contribution, while the higher state contribution will be suppressed.

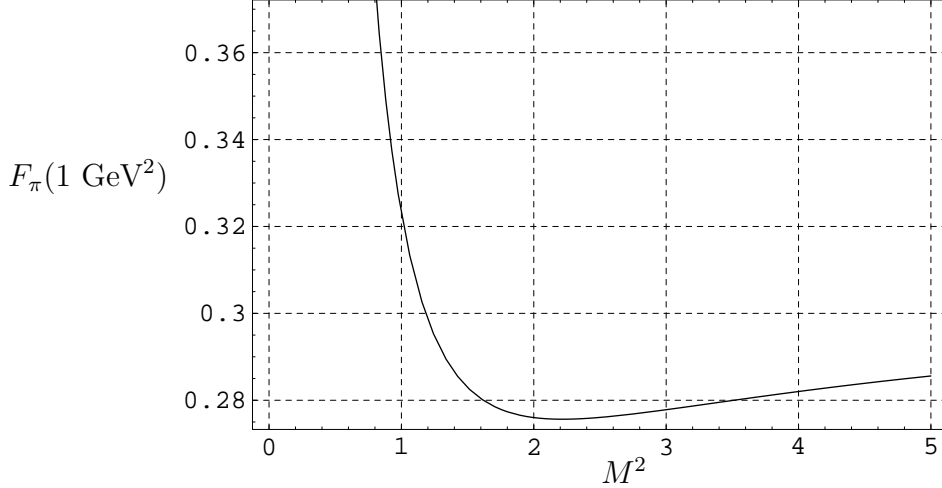


Figure 3: Borel mass  $M^2$  dependence of pion electromagnetic form factor at  $Q^2 = 1 \text{ GeV}^2$

Equating Borel transformed theoretical and physical parts of QCD sum rules we get

$$F_\pi(Q^2) = \frac{4}{f_\pi^2} \left( \Phi(M^2, q^2) + \frac{\alpha_s}{48\pi M^2} \langle 0 | G_{\mu\nu}^a G_{\mu\nu}^a | 0 \rangle + \frac{52\pi}{81M^4} \alpha_s \langle 0 | \bar{\psi}\psi | 0 \rangle^2 \left( 1 + \frac{2Q^2}{13M^2} \right) \right), \quad (38)$$

where

$$\Phi(M^2, q^2) = \frac{1}{(2\pi)^2} \int_0^{s_0} dx \exp \left[ -\frac{x}{M^2} \right] \int_0^x dy \rho^{\text{pert}}(s_1, s_2, q^2). \quad (39)$$

Here, for continuum subtraction we used so called "triangle" model. To verify the stability of our results with respect to the choice of continuum model we checked, that the usual "square" model gives similar predictions for pion electromagnetic form factor provided  $s_0 \sim 1.5s_1$  is chosen<sup>4</sup>. In what follows we use  $s_0 = 0.96 \text{ GeV}^2$  for continuum threshold<sup>5</sup>. This value is in agreement with the continuum threshold  $\approx 0.7 \text{ GeV}^2$  for axial polarization operator used in two-point sum rules. Next, we use two-loop renormalization group running of strong coupling constant with  $\Lambda_{\text{QCD}} = 325 \text{ MeV}$  and fix the scale  $\mu$  of strong coupling constant at 2  $\text{GeV}$ . This choice is in agreement with the discussion presented in [26], where it was argued that in the region of momentum transfers  $Q^2 < 10 \text{ GeV}^2$  the strong coupling constant should be taken

<sup>4</sup>For more information about different continuum subtraction models see [7]

<sup>5</sup>In general, the value of continuum threshold is determined from the ratio of nonperturbative contribution to leading perturbative term in OPE for our correlation function.

at frozen value  $\alpha_s \sim 0.3$ . In Fig. 3 we plotted the dependence of the pion electromagnetic form factor from the value of Borel parameter  $M^2$  at  $Q^2 = 1 \text{ GeV}^2$ . It is seen that "stability plateau" starts to develop for  $M^2 > 2 \text{ GeV}^2$ . To proceed further we could fix the value of Borel parameter at  $M^2 = 2 \text{ GeV}^2$  and get results for pion form factor as a function of momentum transfer. However, doing so we limit the range of  $Q^2$  where our results could be considered as reliable. To see it, it is instructive to investigate large  $Q^2$  limit of pion form factor. In the limit  $Q^2 \rightarrow \infty$  perturbative contribution to pion form factor decreases as  $Q^{-2}$ , while power corrections grow with  $Q^2$ . It turns out, that our sum rules become inapplicable at relatively large momentum transfers  $Q^2 > 4$ . Therefore, to cover all experimentally accessible range of  $Q^2$  for pion electromagnetic form factor we will explore our sum rules in the limit of infinite Borel parameter  $M^2$ . So, basically, here we are employing local duality approach. The local duality approach implies the following relation for continuum threshold [26, 27]:  $s_1 = s_2 = 4\pi^2 f_\pi^2 / (1 + \frac{\alpha_s}{\pi}) = 0.62 \text{ GeV}^2$ , which ensures the Ward identity for pion electromagnetic form factor up to NLO ( $F_\pi(0) = 1$ ). The results for pion electromagnetic form factor are shown in Fig. 4 (solid line is the sum of LO and NLO contributions, curve with long dashes denotes LO contribution and curve with short dashes stands for NLO contributions.).

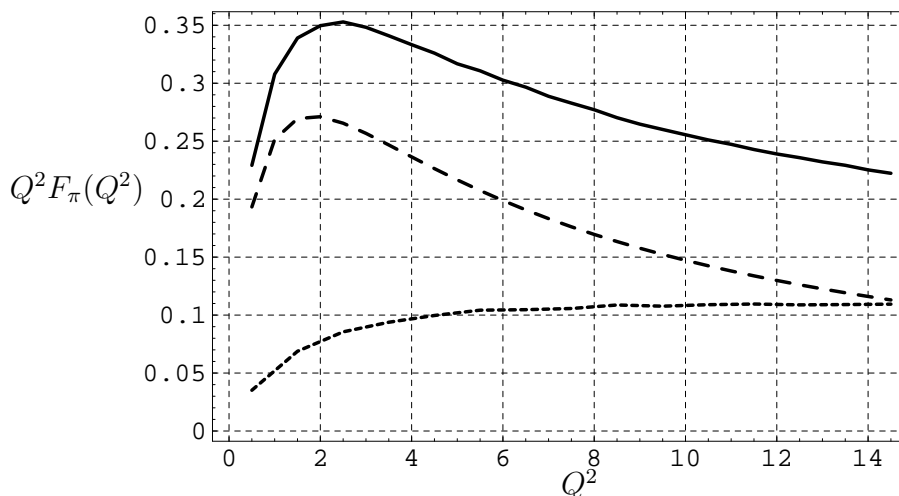


Figure 4:  $Q^2$  dependence of pion electromagnetic form factor

Now, let us recall that in the limit  $Q^2 \rightarrow \infty$  our QCD sum rule prediction coincides with LO pQCD results with asymptotic expression for pion distribution function. We could improve our prediction further with inclusion of NLO pQCD corrections.

Within perturbative QCD pion electromagnetic form factor is given by the following factorized expression

$$F_\pi^{\text{pQCD}} = \int_0^1 dx \int_0^1 dy \phi_\pi(x, \mu) T_H(x, y, Q^2, \mu^2) \phi_\pi(y, \mu), \quad (40)$$

with

$$T_H(x, y, Q^2, \mu^2) = \frac{2\pi C_F \alpha_s(\mu) f_\pi^2}{N_c Q^2 (1-x)(1-y)} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} T_1(x, y, Q^2/\mu^2) + \dots \right] \quad (41)$$

It should be noted that this contribution contains explicit dependence on pion wave function and gives us a possibility for its determination through the comparison between combined predictions

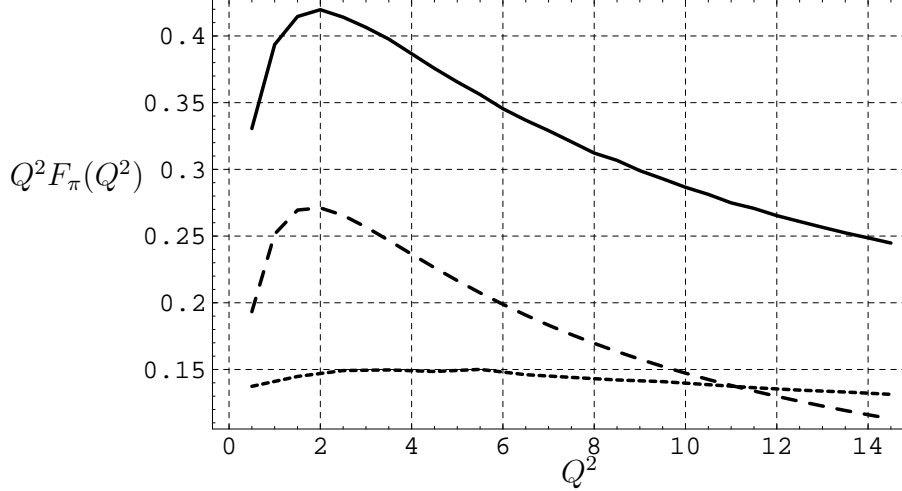


Figure 5:  $Q^2$  dependence of pion electromagnetic form factor including NLO pQCD corrections

of QCD sum rules and pQCD frameworks with experimental data. We suppose to perform this analysis in future, while in present work we limit ourselves only to the inclusion of NLO pQCD prediction with the use of asymptotic pion distribution functions. Within this approximation pQCD predictions for pion electromagnetic form factor are given by [14, 15, 16, 17, 18] :

$$F_{\pi}^{\text{pQCD}}(Q^2) = F^{\text{LO}}(Q^2) + F^{\text{NLO}}(Q^2), \quad (42)$$

where

$$F^{\text{LO}}(Q^2) = 8\pi\alpha_s(\mu^2) \frac{f_{\pi}^2}{Q^2}, \quad (43)$$

$$F^{\text{NLO}}(Q^2) = 8 \frac{f_{\pi}^2}{Q^2} \alpha_s^2(\mu^2) \left[ 6.58 + \frac{9}{4} \log \left( \frac{\mu^2}{Q^2} \right) \right]. \quad (44)$$

The results for pion electromagnetic form factor including NLO pQCD corrections are shown in Fig. 4 (solid line is the sum of QCD sum rule and NLO pQCD contributions, curve with long dashes denotes LO QCD sum rule contribution and curve with short dashes stands for the sum of NLO QCD sum rule and NLO pQCD contributions.). We see, that obtained results for pion electromagnetic form factor are in good agreement with available experimental data as well as already available theoretical estimates performed with the same level of precision.

## 4 Conclusion

We presented the results for pion electromagnetic form factor in the framework of three-point NLO QCD sum rules with axial interpolating currents for pions. To improve our estimates we also took into account NLO pQCD contributions with asymptotic pion distribution functions. The theoretical curve obtained for  $Q^2$  dependence of pion form factor is in a good agreement with existing experimental data. Here, we for the first time computed radiative corrections to double spectral density entering three-point sum rules with axial currents. These corrections turned out to be large and should be taken into account in rigorous analysis.

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